

# ***Application of Neural Networks to Stock Market Prediction***

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## Introduction

The aim of this project is to predict the future values of the Standard & Poor's 500 (S&P 500) stock index based on its past values and the past values of some financial indicators. There have been many attempts at predicting stock market movements, most of them based on statistical time series models [1]. Most of these attempts have been unsuccessful due to the complex dynamics of the stock market. The *efficient market hypothesis* says that stock prices rapidly adjust to new information by the time the information becomes public knowledge, so that prediction of stock market movements is impossible [2]. This hypothesis seems to be correct for static and linear relationships explored traditionally using multiple regression analysis. However, it is possible that dynamic and non-linear relationships exist which cannot be modeled by traditional time series analysis methods [3]. This, is the motivation for application of neural networks to financial time series analysis.

A huge amount of research is being done on the application of neural networks to stock markets. Some of the applications include prediction of IBM daily stock prices [4], a trading system based on prediction of the daily S&P 500 index [5], short term trend prediction using dual-module networks [6], weekly index prediction [7], monthly index prediction using radial basis functions [8] etc. Some of these papers use the past values of the stock index only, as the input to the neural network so as to obtain the future values, while some use additional fundamental and financial factors as inputs.

This project explores the effect that short and long term interest rates have on the stock market, in particular on the S&P 500 index. It is well known that an increase in interest rates tends to lower the stock market and vice versa [9]. The hypothesis is that the current stock prices indicate the cumulative sum of the present and future worth of any company. If the interest rates increase, the equivalent future value of a stock in terms of today's dollars reduces, causing the stock price to reduce. Financial experts report that the long term value of a broad based index is affected by interest rates after some unknown delay. However, the exact effect is unknown and hence, a neural network can be suitably applied to find this non-linear mapping. The next section is a brief review of some of the similar work done while section three describes the selection of features from the raw data for training of the neural network. Test results for different strategies are given in section four and the last section lists the conclusions from this project.

## 2. Literature review

The work done in the area of **stock market prediction using neural networks** can be classified into two broad categories:

- Prediction using past stock index values and momentum indicators based on these values, and
- Prediction using past stock index values and other fundamental factors such as interest rates, foreign exchange rates, up and down volumes, bond rate etc.

### ***Prediction using past stock values only***

This **first category** is based on the theory that all the information available regarding any economic indicators is already contained in the time history of the index and future index values are dependent on data upto the present moment. Some applications of this category are discussed below.

#### **Dual module neural network**

One application [6] uses a dual module neural network, with one module learning the long term trend of the market and the other module trained to learn the short term rate. The data used is a 4-tuple of index values, namely the high, low and closing values and trading volume for each day. By taking moving averages and variances, this set of 4-tuples for all days are combined into a 16-tuple which is level insensitive and time-invariant. These extracted features are then used to train the two networks. The long term net uses a training window spanning the last trading quarter while a 12-day window is used for the short term module. The authors conclude that the neural network shows a much better response than multiple linear regression.

#### **Neural sequential associator**

In this paper [10], the author uses a feedforward neural network with the last  $n$  stock index values as inputs and the next  $N-n$  values as the outputs. This is a  $N-n$  step ahead prediction. Thus, if index for the  $n^{\text{th}}$  day is denoted by  $X_n$ , then, the inputs are  $X_1, X_2, \dots, X_n$  and the outputs are  $X_{n+1}, X_{n+2}, \dots, X_N$ . If such a network is trained, any correlation between the index values for the  $n+1$  through  $N^{\text{th}}$  day will be neglected. To ensure that this does not happen, the network is trained with errors between the desired and actual outputs in addition to the  $n$  inputs. These errors will then be  $(X_{n+1} - Y_{n+1}) \dots$ , where  $Y$  is the output of the network. As training proceeds this error will tend to zero and these additional inputs are not required in the testing phase. This work also uses two neural networks, one to learn the global features and another to learn the local features or small fluctuations.

#### **Recurrent neural network approach**

Kamijo and Tanigawa [11] propose the use of an *Elman recurrent net* for predicting the future stock prices using extracted features from past daily high, low and closing stock prices. The method used tries to extract triangle patterns in stock prices which are seen graphically by plotting the daily high, low and closing prices. A *triangle* refers to the beginning of a sudden stock price rise after which the high and low prices appear and the price oscillates for some period before the lines converge. The neural network is trained to recognize such *triangle* patterns in the stock prices. As such, it is mainly a categorization approach to recognize whether a pattern is a *triangle or not*. Such knowledge can be useful in judging whether a price rise is permanent.

### ***Prediction based on past stock values and other fundamental indicators***

The **second category** of research assumes that other fundamental factors such as present interest rates, bond rate and foreign exchange rates affect the future stock prices. Since,

this is also the focus of the present project, some of these papers are described in greater detail especially with respect to the feature extraction, wherever such information has been made available in these publications.

### **Modular neural network approach**

Kimoto et al [12] use several neural networks trained to learn the relationships between past values of various technical and economic indices for obtaining the expected returns of the TOPIX. The TOPIX is a weighted average of all stocks listed on the Tokyo Stock Exchange and is similar to the Dow Jones Industrial Average (DJIA). The technical and economic indices used are: the vector curve (an indicator of market momentum), turnover, interest rate, foreign exchange rate and the DJIA value. The desired output of the networks is a weighted sum, over a few weeks, of the logarithm of the ratio of the TOPIX at the end of week  $t$ , to the TOPIX value at the end of week  $(t-1)$ .

Thus,  $\mathbf{r}_t = \ln(\text{TOPIX}(t)/\text{TOPIX}(t-1))$  and the desired output is a weighted sum of  $\mathbf{r}_t$  for some weeks. The feature extraction is not explained in [12] except for the fact that some irregularity is removed and logarithm function is used before normalization. The authors claim that the use of the weighted sum of the outputs of many neural networks reduces the error, especially since the returns are predicted for a few weeks. A *buy/sell* system is setup based on the predicted returns and this system is shown to perform much better than a *buy-hold* strategy. However, the teaching data uses future returns, so that this method cannot be used for actual stock trading. (The authors do mention this as part of their proposed future work).

### **One week ahead prediction using feedforward networks**

This work [7] uses a simple feedforward neural network trained using past and present data to predict the value of the FAZ-Index which is the German equivalent of the DJIA. Input data includes the moving average of past 5 and 10 weeks of the FAZ-Index, a first order difference of the FAZ-Index and its moving averages, the present bond market index and its first order difference and the Dollar-Mark exchange rate along with its first order difference. The value of the FAZ index is predicted for the next week based on this data. The network is trained using data for the past  $M$  weeks and is then tested based on data for the next  $L$  weeks, where  $M$  is called the training window and  $L$  is called the testing window. For successive prediction, the windows are moved ahead and the network is retrained. Three different networks are compared each having a different set of inputs, one of which has only the last 10 FAZ index values as the input. It is seen that the network fed with technical indicator data performs better than the one trained only on past index values. Normalization of training data is done so as to keep the data within 0.1 and 0.9, however, the normalization method is not given. *This approach is particularly suitable to the aim of this project and is hence, used as the basic method in this project with certain modifications.*

### **Radial basis function approach**

Komo, Chang and Ko [8] use a **radial basis function** network to predict the future stock index values based on past data of the index and other technical indicators such as transaction costs, bond market values and futures prices. The RBF network has two

hidden layers, with the first hidden layer being trained using the K-means clustering algorithm which is thus, an unsupervised learning algorithm. Once an initial solution for the means and standard deviations of each neuron is found using the clustering algorithm, a supervised learning algorithm is applied to fine tune the parameters of both the hidden layers. Gradient descent is used for the supervised learning. No details are given regarding the inputs used or the feature extraction.

### **Multi-component trading system using S&P 500 prediction**

Obradovic et al [5] use two neural networks, to predict the returns on S&P 500 stock index. One network is trained using upward trending data while another is trained for downward trending data. The test data is fed to both the neural networks after filtering. A complex filtering scheme is used based on traditional direction indicators. The outputs of the two networks are combined using a high level decision rule base to obtain buy/sell recommendations. The inputs used are the S&P 500 index return for the past 3 days and the US Treasury rate lagged 2 and 3 months. The authors report that a simple filtering scheme gives better results than the more complex scheme using directional indicators. The average annual rate of return obtained using this approach is close to 15% which is much higher than the return obtained using a buy-hold strategy. This approach is not used in this project owing to its complexity.

Some of the literature dealing with stock market prediction is described above. Of these papers, the present project is based on the **one week ahead prediction approach** of [7]. The next section describes the assumptions underlying the work done in this project, the analysis of data and extraction of features for training of the neural network.

## **3. Data Analysis and Feature extraction**

The basis of this project is the assumption that in the long term, interest rates affect the stock market after some delay. There are many models constructed by economic analysts to prove such a correlation. One of the simplest models is that proposed by Martin Zweig in [9], where some fundamental indicators such as the prime lending rate, the Federal reserve lending rate and the consumer price index as well some technical indicators such as the up/down volume ratio, bullish/bearish index based on newspaper advertisements and other momentum indicators are combined in a super model. The output of the supermodel is in terms of percent points which is used to obtain *buy/sell* signals so as to time the market *in the long run*. The model is shown to be able to predict most of the big bull and bear markets over the past years. Construction of such a model requires some amount of experience in dealing with the stock market, although once it is constructed, the functioning can be autonomous.

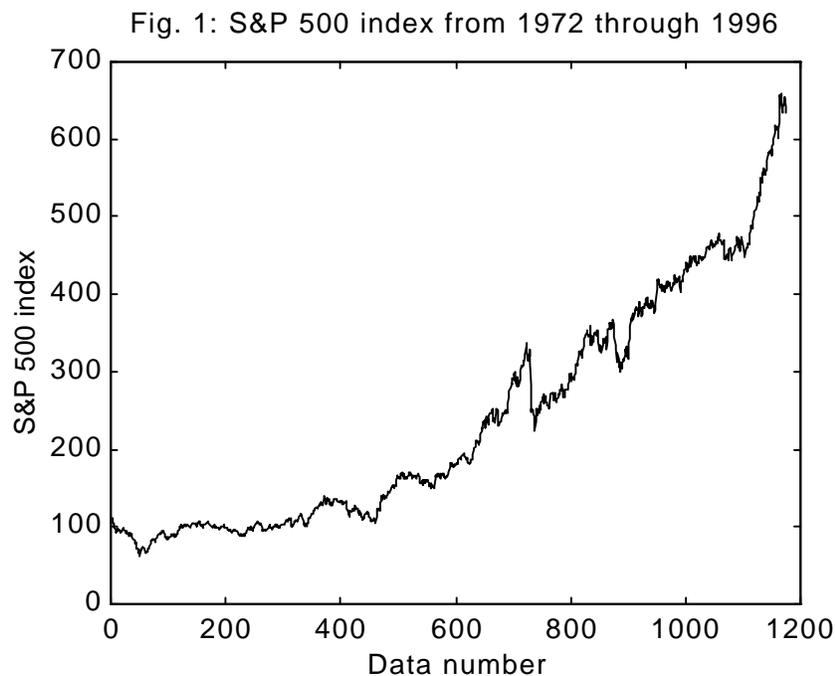
In this project, an attempt is made to construct a neural network based model. The input data, which was kindly made available by Prof. Robert Porter includes the *short term (3-month) interest rate*, the *long term (10-year) interest rate* charged by banks and the *Standard and Poor's 500 stock index (S&P 500)* from October 1972 through April, 1996. In addition, the *consumer price index (CRBX)* is also available from December, 1986

through January, 1996. However, the *CRBX* data is not used for this project. **The help of Prof. Porter from the University of Washington's Applied Physics Laboratory in making the data readily available is gratefully acknowledged.**

### **Data Analysis**

It is proven as seen from the performance of the Zweig model that a rise in interest rates *generally* reduces stock prices and vice versa. This may not hold true if the momentum of the market opposes the effect of the interest rate change. To analyze the effect of interest rates, the cross-correlation between the long term interest rate, delayed by 53 weeks and the *S&P500 index* was obtained using *xcorr()* in MATLAB. Since, the whole time series was used, the results gave the cross-correlation for various delays. Similarly, the cross-correlation between the short-term interest rate and the index is also obtained.

To get some logical output from such a calculation, the *S&P500 index* was detrended. The plot of the index from 1972 through 1996 is shown in Fig. 1 below.



The trend is clearly exponential, which is logical since inflation cumulatively reduces the value of the dollar. To detrend this data, the logarithm of this data was taken and this logarithm of the index was detrended using *detrend()* in MATLAB, using a line detrending, rather than simply removing the mean. Similarly, the long and short term interest rates were detrended by simply removing the mean. Fig. 2 and 3 show the cross-correlation between the long term interest rate delayed 53 weeks and the detrended index (Fig. 3 is a zoomed in version of Fig. 2). It can be seen that there is a strong negative correlation between the long term rate delayed 7 weeks through 25 weeks. Although, the correlation with the lesser delay is stronger, here the long term rate delayed by 25 weeks is used.

Fig. 2: Correlation - long term rate and index

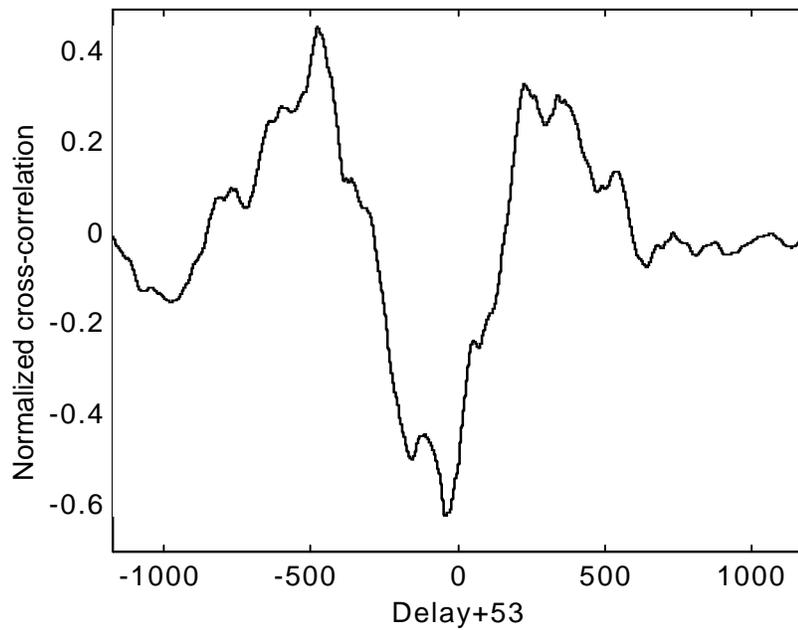
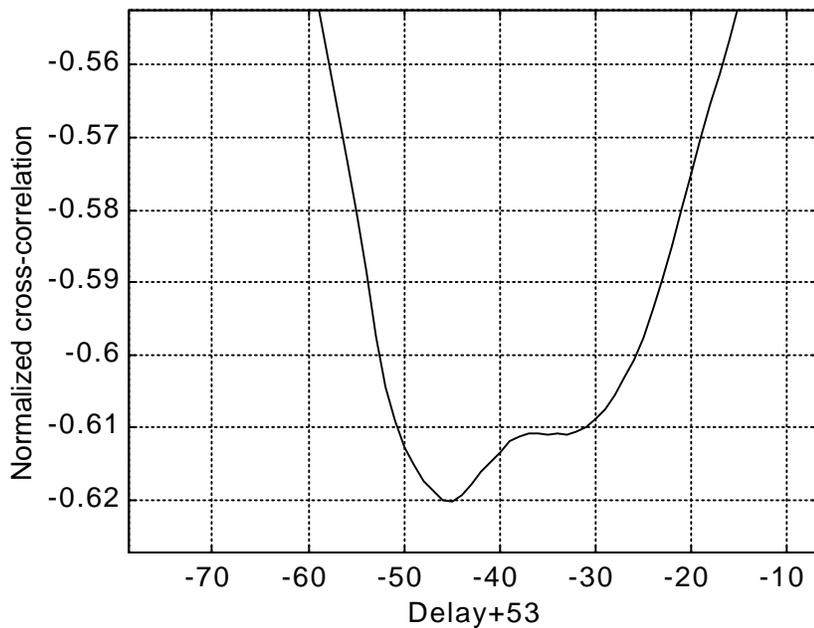


Fig. 3: Correlation - long term rate and index



Similarly, Fig. 4 and 5 show the complete and zoomed in cross-correlation's between the short term interest rate and the index. From Fig. 4 and 5, it can be seen that there is a strong negative cross-correlation between the short term rate and the index, with the short term rate lagging the index either by 20 weeks or by 48 weeks. Since, both these inputs are sufficiently delayed from the index, both are used as inputs to the neural network.

Fig. 4: Correlation - short term rate and index

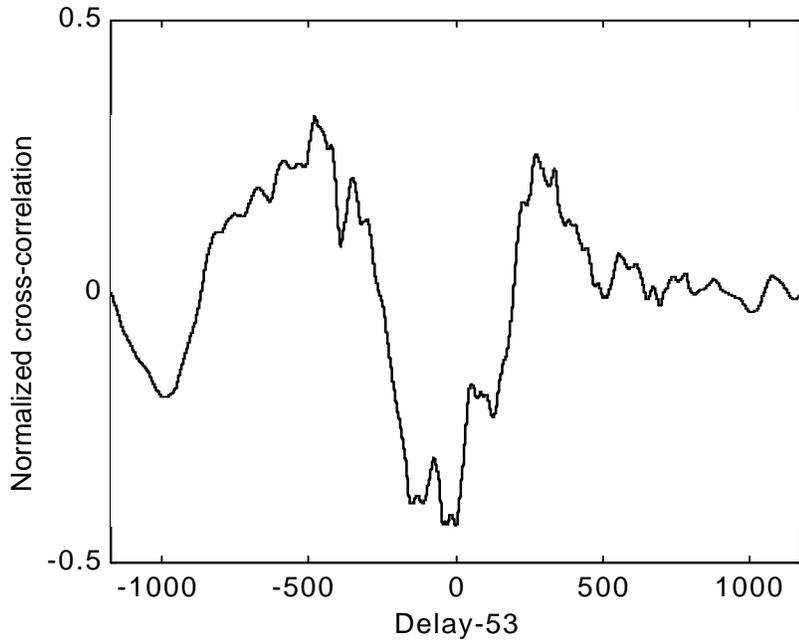
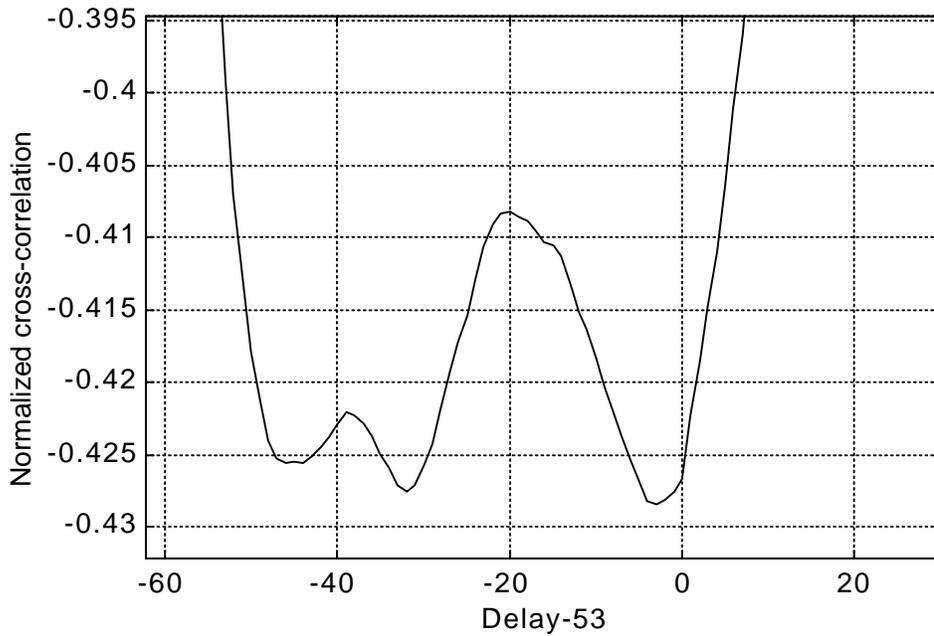


Fig. 5: Correlation - short term rate and index



### **Feature extraction**

Given, the above analysis the inputs to the neural network are:

- The stock index value for the present week (all values are weekly closing values)
- the weekly long term interest rate, delayed by 25 weeks,
- the weekly short term interest rate, delayed by 20 weeks and
- the weekly short term interest rate, delayed by 48 weeks.

The stock market future value also depends on the trend of the market as well as the trend of the interest rates. To include this information, additional inputs are used. These are:

- The difference between the present week's index and that of the previous week (referred to as the first difference). Since, this difference can be positive or negative, it is encoded (as done in [7]) using two inputs. The first one is the absolute value of this first difference and the second one is arbitrarily chosen as: *0.8 if the difference is positive, 0.2 if the difference is negative and 0 if there is no change*. This encoded sign of the difference is hereafter referred as the *trend*. *These two inputs along with the present S&P500 index value make up the first 3 inputs of the network*.
- To reduce the effect of the noise present in the weekly stock market variation, the average of the last 5 weekly closing values of the index and the average of the last 10 weekly closing values of the index, along with their absolute first differences and trends are used. *These constitute the next 6 inputs of the network*.
- To account for the long term interest rate, the long term interest rate delayed by 25 weeks and its absolute difference and trend are used as *inputs 10 through 12*. The short term interest rate is accounted for by using the rate delayed by 48 weeks as well as the rate delayed by 20 weeks along with their respective absolute differences and trends, thus giving *inputs 13 through 18*.

The desired output of the network is the stock index value for the next week. Thus, the neural network to be used has *18 inputs and 1 output*. A simple feedforward network is used with standard vanilla backpropagation training.

### **Normalization**

The stock market index as seen from Fig. 1 has a continuously rising exponential trend. If the index value is normalized linearly and fed to the neural network, it is possible that the normalized future values of the index will be greater than unity, so that the neural network will never be able to track those values. Hence, a **non-linear normalization** is required. The function selected for normalization is the *hyperbolic tangent* function, as suggested in [10]. The actual function used is:

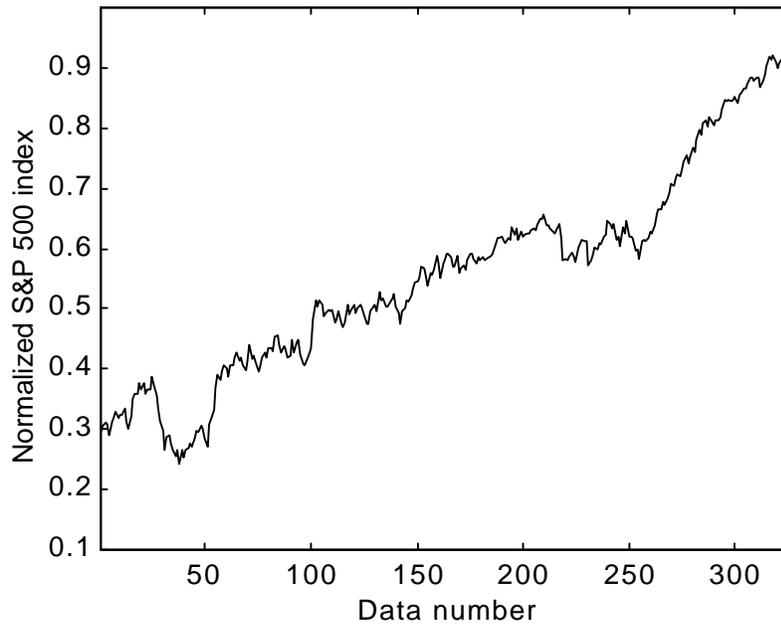
$$\text{spx\_norm} = 0.5 * (1 + \tanh((\text{spx} - \text{spx\_avg}) / A)) \quad (1)$$

where, *spx\_norm* is the normalized value and *spx\_avg* is the average of the index over the **training data only**. *A* is a constant to obtain the desired data range (so as to avoid saturation of the *tanh* function. The *tanh* function will give an output in the range [-1,1], which is then converted to [0,1] linearly (by adding 1 and then multiplying by 0.5).

As a first try, such a normalization was used and the data selected was the last 325 weeks (6 years) with the testing data being the last 50 weeks and the training based on 275 weeks before the testing data. As, seen from Fig. 1, the stock index is higher for the test data than for the training data, so that the neural network never sees the desired values in the range of the test data during training. This can be seen from Fig. 6, which shows the normalized value for the training and testing data of the *S&P500 index*. In the first 275 data points, which constitutes the training set, the normalized index has a maximum value of 0.7339, while in the test region the maximum value is 0.9211. Thus, the network cannot

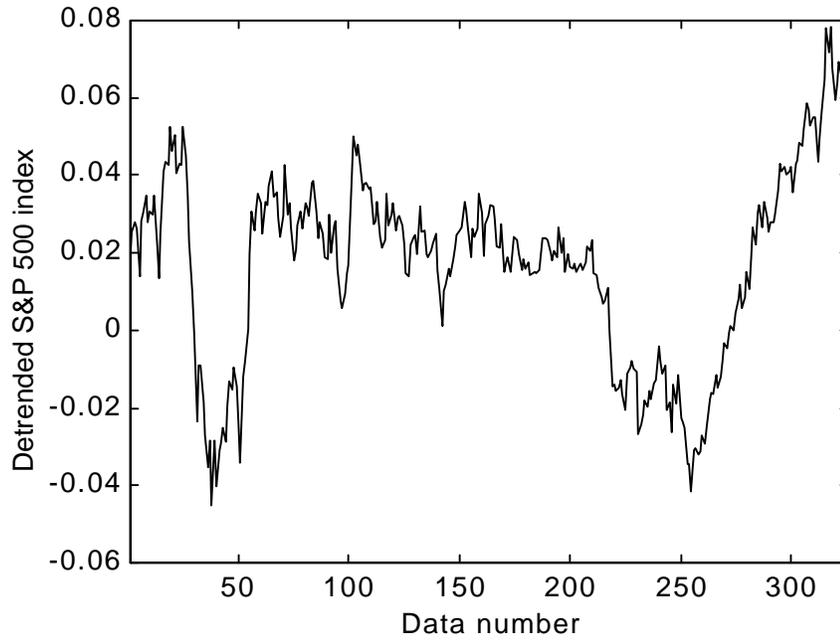
be expected to learn the relationship and give the correct output and this was confirmed after training and testing of the network using this kind of normalization.

Fig. 6: Normalized index in training and test regions



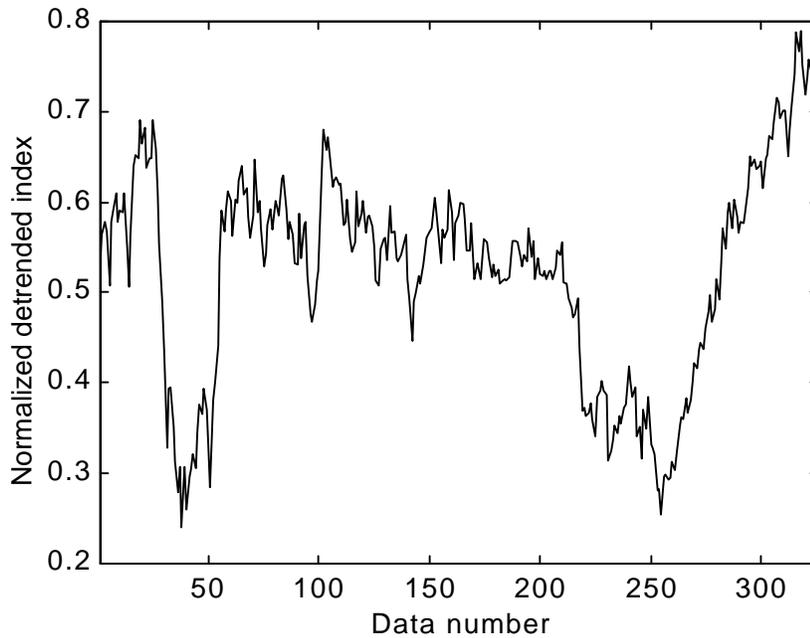
To remove this problem, the solution chosen was to *detrend the data* by taking the logarithm of the *S&P 500 index* and then removing the linear trend from it. The detrending was done by fitting a line (using *polyfit()* in MATLAB) to the **training data only** and then removing this line from both the training and testing data. Fig. 7 shows the detrended combined training and testing range of the *S&P500 index* using such an approach. Although, the desired output is still higher at the end of the testing region, the difference is much less than in Fig. 6.

Fig. 7: Detrended index in training and test regions



This detrended index is then used to obtain the 5-week and 10-week moving averages as well as the *first differences* and *the trends*. The detrended index is then normalized using equation (1), with  $A = 0.1$  and this normalized detrended index is plotted in Fig. 8, below.

Fig. 8: Normalized detrended S&P 500 index



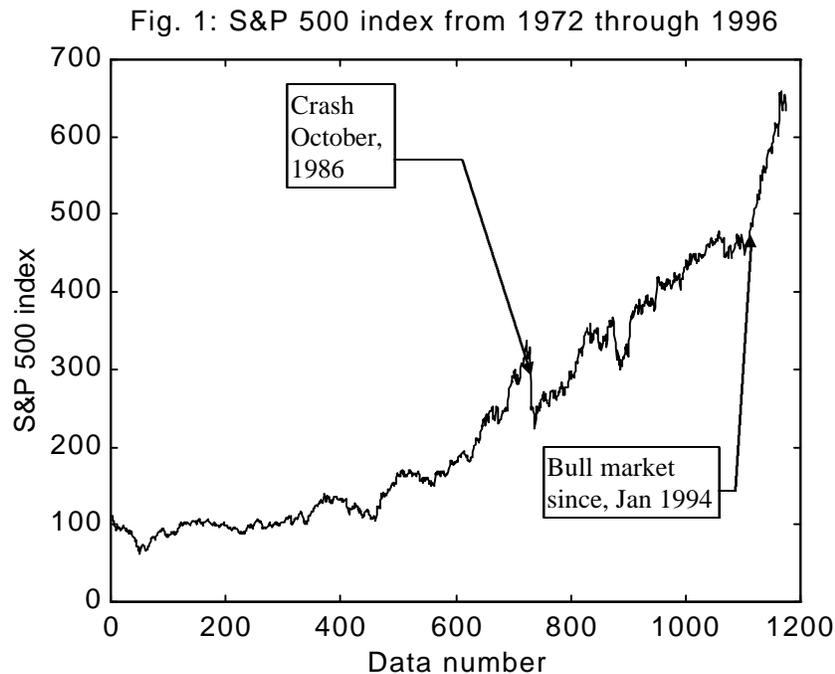
Given this data, the network can be expected to learn the trend and give good results. The 5-week and 10-week averages of the detrended index as well as the absolute value of the first difference of the detrended index and averages are all normalized using eq. (1), with

different values for  $A$  ( $A = 0.1$  for the index and the averages and  $is = 0.02$  for the absolute value of the 3 first order differences).

The interest rates fluctuate about a certain value and there is no cumulative or time integral effect on their behavior, hence, the interest rates are not detrended and these are normalized using simple linear normalization. The same applies for the absolute difference of the interest rates. All the *trends* have a value of either 0.8, 0.2 or 0.0 and as such don't need further normalization. This completes the data normalization for the neural network. The same techniques and constants are used for any other range of training and testing data.

#### 4. Test results

The neural network was trained using standard backpropagation for 2 different test cases, which are of critical interest from the point of view of desired performance. Fig. 1 shows the variation of the *S&P 500 index* and is reproduced below for ready reference. It can be seen that there is a big market crash around the 730<sup>th</sup> data point (October, 10<sup>th</sup>, 1986) and there is a strong bull market beginning around the 1110<sup>th</sup> data point (January, 21<sup>st</sup>, 1994). It would be interesting to see how the network performs for these worst case examples, since, it would be expected to perform extremely well, when the training and testing data is in the same range.



To test the network in these critical regions, it is trained using data from about 4 years prior to the crash of 1986 or the start of the 1994 bull run. For both these test cases, the network used has one hidden layer with 7 hidden neurons and 19 input neurons (input data plus a unity input for the offset). Weights are initialized to random values in the range  $[-1,1]$ . The hidden and output layers use a sigmoidal activation function. While the offset for the hidden layer is provided by the fixed unity input of the 19<sup>th</sup> input neuron, there is no

offset for the sigmoids in the output layer. These test cases and the results obtained are analyzed below.

### Case 1: Bull run since Jan, 1994

Fig. 9 shows the normalized training and test data of the *S&P500 index*, while Fig. 10 shows the normalized training and test data for the interest rates. The data starts from the 850<sup>th</sup> data point and the training data is of length 275 while the test data has a length of 50 weeks. It can be seen that the long term interest rate is rising for most part of the bull run (testing data) except at the end, while the short term rate delayed by 20 weeks is rising initially and then constant. This change from rising to constant is expected to have some effect on the market. However, in this region the main effect is expected to come from the market momentum itself, as analyzed by economists elsewhere [9].

### Test results

Fig. 11 shows the test results for this case. The testing is done using the trained network and the output for each week is compared with the desired output. It can be seen from the figure that the network is able to predict the bull run consistently, one week in advance. It predicts the correct trend of the stock index 43 times out of the test sample of 50 points. The maximum percentage error expressed as a percentage of the desired output is only 4.044 %, while the average error is only 0.95%. Thus, the networks performs extremely well and this performance can be suitably used to time the market.

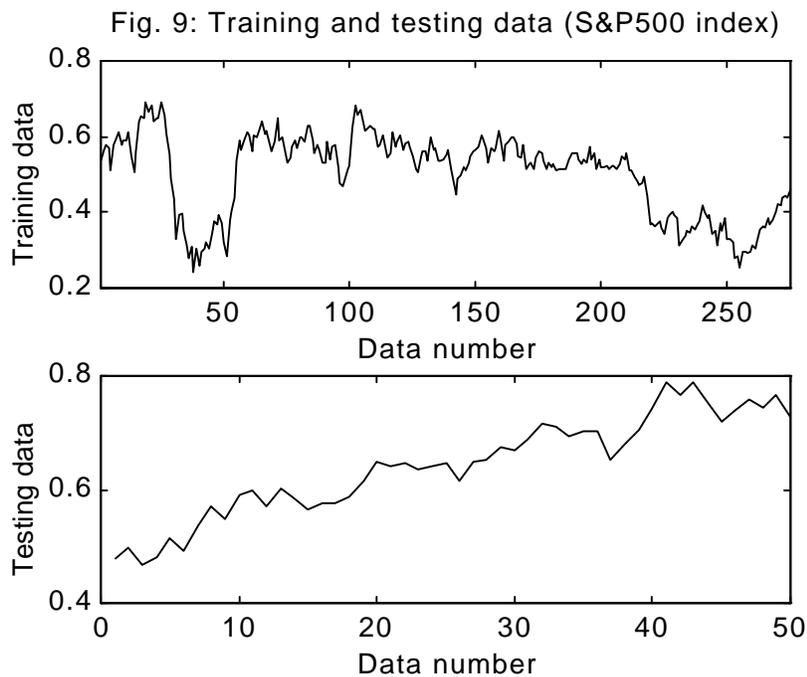
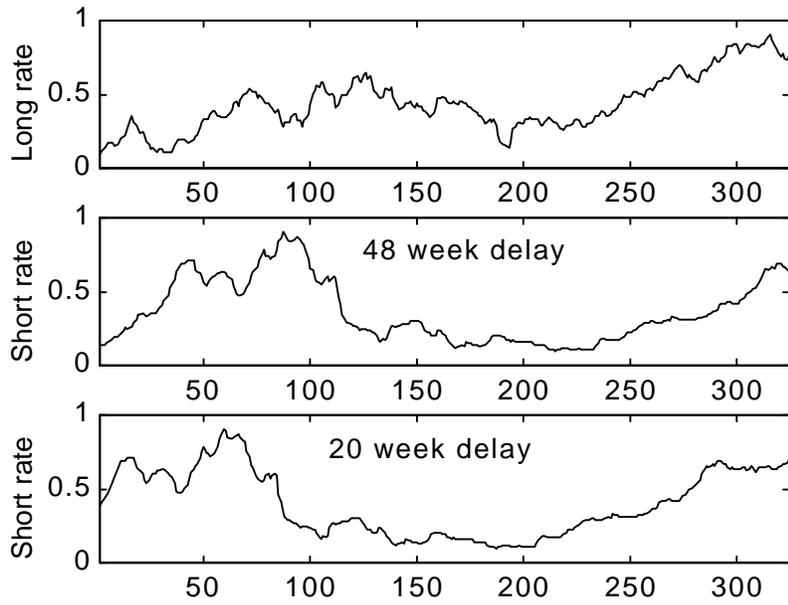
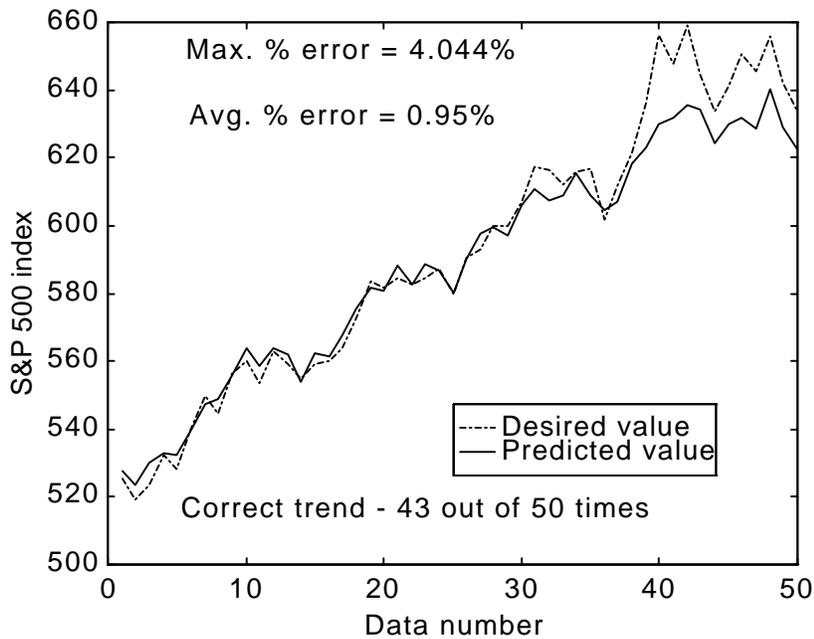


Fig 10: Training and test data for interest rates



Even after detrending and normalization, which is based on **training data only**, the desired output from the network is higher than the values it is trained on (Fig. 9). As such, the network is able to predict an **increase** in the output even if it is not trained for the exact values of the output range. This augurs well for a good performance in the future, provided that care is taken to normalize the data so that a sudden increase in the index value will not saturate the normalized value.

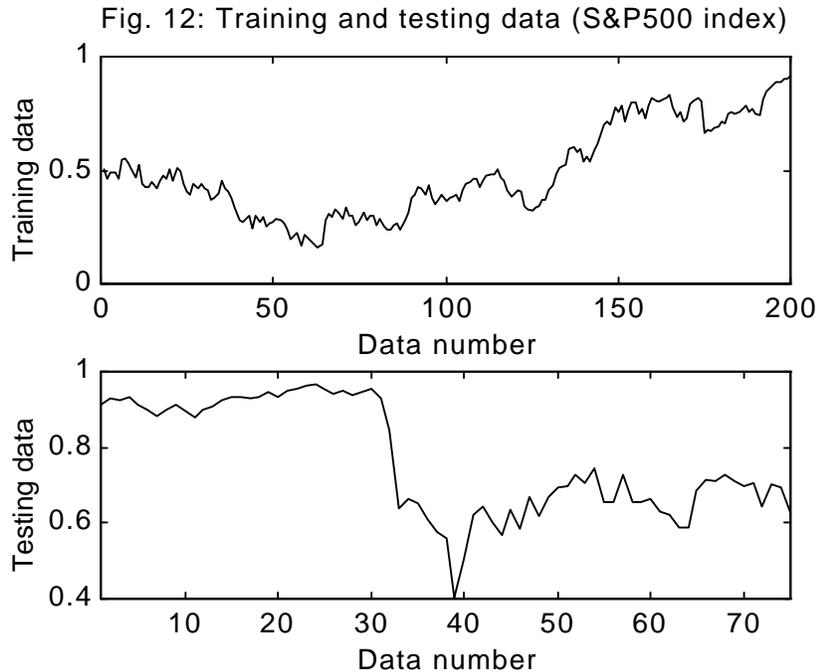
Fig. 11: Predicted and desired S&P 500 index

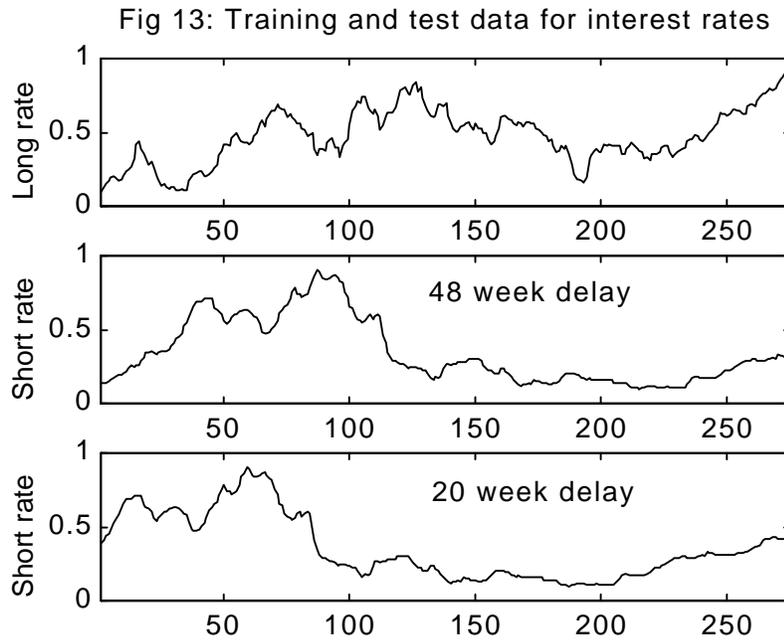


It can be argued that the network can be trained every week rather than keeping it based on the training, which will be very old near the end of the 50<sup>th</sup> week. Moving the training window every week and retraining the network is a valid approach, which might be necessary in practice. However, there is a danger of the network training on the noise, inherent in the weekly changes and hence, performing worse than this network. In any case, this procedure can be modified suitably and the prediction window can also be reduced to suit the requirements.

### Case 2: Crash of October, 1986

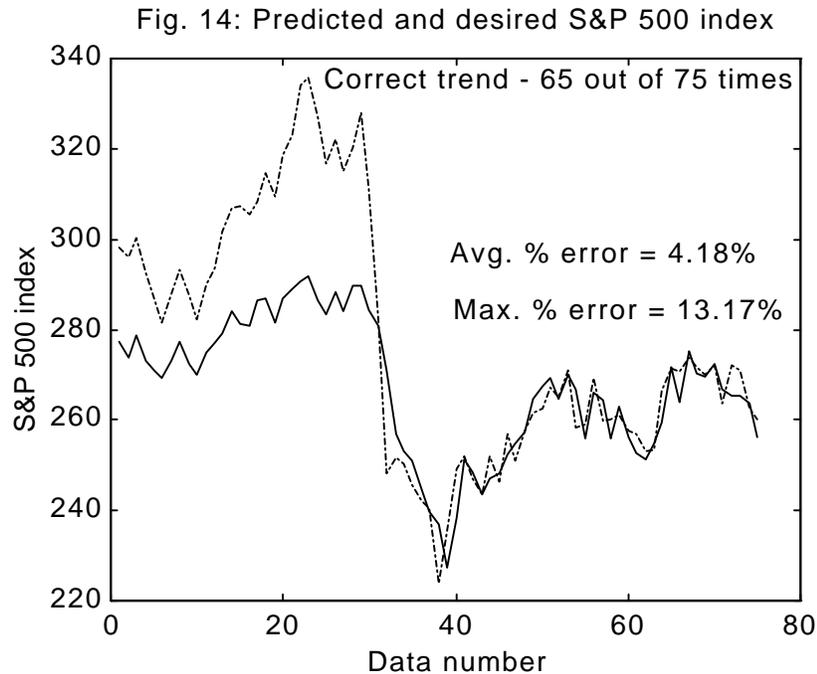
Fig. 12 on the shows the index values in the training and testing region for this second case, where the training data starts from the 500<sup>th</sup> data point and is 200 data points long while the test data extends over 75 weeks immediately following the training data and includes the big crash. Fig. 13 shows the interest rates for the training and test regions combined. In this case, the delayed long term interest rate is falling in the training phase when the *S&P 500 index* has a rising trend, while during the testing phase it is rising very fast, when the index is falling. The short-term rate delayed by 20 weeks is also increasing, albeit rather slowly, in the test period. Thus, in this test case, it is expected that the effect of rising interest rates will predict the crash. There is not much sustained momentum in the training data of the index, since it is rising and falling or in other words, the change in index is oscillating between positive and negative values. The network having 7 neurons in the hidden layer is trained for 900 iterations, with a constant step size of 0.4 and the test results are detailed in the next section.





### Test results

Fig. 14 shows the test results for this case. The plot shows that the network is able to predict the severe crash **one-week** before it occurred. Although, the network output is lower than the actual values in the initial part of the test range, there is no trend in the stock index, which would force the network to predict a crash. Thus, a network using only the past index values would have been unable to predict this crash, which can be attributed to the rising long term interest rates. For this case, the network predicts the correct trend of the stock index 65 times out of the test sample of 75 points. The maximum percentage error expressed as a percentage of the desired output is much higher than case 1 and is 13.17 %, while the average error is only 4.18%. The main source of the error is that the network doesn't predict the initial rise in the market very well, in terms of the index level. This is in fact conservative since, any other model which would have predicted the initial high rise correctly might have given an incorrect and premature buy signal immediately followed by a sell signal. Such rapid changes are not desirable since the number of trades is increased along with the associated transaction costs.



This concludes the presentation of the test results for both the cases. The next section presents the conclusions and possibilities of future work based on this project.

## 5. Conclusions and Future work

The following conclusions can be drawn from this project work:

- A feedforward neural network has been successfully applied to the problem of a **one-week ahead prediction of the weekly closing prices of the S&P 500 index**.
- The dependence of the stock index on interest rates is established using cross-correlation values and the inclusion of this dependence is seen by the performance of the network for the second test case.
- The trained neural network performs very well even for worst cases where there are sudden rises or falls in the stock market index.

### **Future work**

The original aim of this project was the application of the available data for **medium term** prediction of the stock market index using some form of a recurrent network.

- For medium term prediction, the method used in [10] and described in Section 2, can be applied. There, the network predicts the stock value for the next few weeks and the training for that is performed by feeding the error for outputs back as inputs during training.
- A recurrent network such as an Elman net can be applied so that the effect of the interest rates on the stock index prior to the data used can also be accounted for. Thus, a history or integral effect can be obtained using a recurrent network.
- Even if a recurrent network is used, some delay will have to be assumed for using the interest rates. The easiest case would be to use current interest rates without any delay

and allow the training to establish the actual delay. There is a lot of scope for work with regard to the choice of the delay in the interest rates.

- For using this technique in practice, a trading mechanism based on the predictions of the network needs to be established which will give *buy/sell* signals and will maximize profit without increasing the number of trades by a large number.

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